Concentration analysis of multivariate elliptic diffusions

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Some known concentration results for Markov processes

Let X be a nice ergodic Markov processes with semigroup $(P_t)_{t\geqslant 0}$, invariant distribution μ and generator L on $\mathbb{L}^2(\mu)$ (endowed with inner product $\langle f, g\rangle_\mu = \int fg \,\mathrm{d}\mu$) and denote

$$\mathbb{C}_\nu(f, T, x) := \mathbb{P}^\nu\left(\left|\frac{1}{T}\int_0^T f(X_t)\,\mathrm{d}t - \mu(f)\right| > x\right), \quad f \in \mathbb{L}^2(\mu), x, T > 0.$$

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Bounds have been mostly studied with two approaches (*Lyapunov vs. Poincaré* [BCG08]):

1. Functional inequalities:
    - Poincaré inequality (PI):

$$\mathrm{Var}_\mu(g) \coloneqq \mu(g^2) - \mu(g)^2 \leqslant -C_P\langle Lg, g\rangle_\mu, \quad g \in D(L).$$

Implies: $\|P_t f - \mu(f)\|_{\mathbb{L}^2(\mu)} \leqslant \mathrm{e}^{-2t/C_P}\|f - \mu(f)\|_{\mathbb{L}^2(\mu)}$

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   - log-Sobolev inequality (LS): $(P_t)_{t\geqslant 0}$ symmetric and
   $$\mathrm{Ent}_\mu(g^2) \coloneqq \mu(g^2 \log g^2) - \mu(g^2)\log\mu(g^2) \leqslant 2C_{\mathrm{LS}}\|\sqrt{-L}g\|^2, \quad g \in D(\sqrt{-L}),$$

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2. Mixing assumptions:
   $$(\alpha(\nu,\varphi)): \quad \alpha_\nu(t) \coloneqq \sup_{s\geqslant 0}\ \sup_{A\in\sigma(X_u, u\leqslant s), B\in\sigma(X_u, u\geqslant s+t)} |\mathbb{P}^\nu(A\cap B) - \mathbb{P}^\nu(A)\mathbb{P}^\nu(B)| \leqslant \varphi(t) \underset{t\to\infty}{\longrightarrow} 0.$$
   For reasonable ν implied by ergodicity of P_t, i.e., $\|P_t(x,\cdot) - \mu\|_{\mathrm{TV}} \leqslant CV(x)\varphi(t)$

Some known concentration results for Markov processes

[Lez01] Suppose $\nu \ll \mu$, $\mathrm{d}\nu/\mathrm{d}\mu \in \mathbb{L}^2(\mu)$ and $\|f\|_\infty < \infty$. If μ satisfies (PI) then we have the Bernstein inequality (BI)

$$\mathbb{C}_\nu(f, T, x) \leqslant 2\Big\|\frac{\mathrm{d}\nu}{\mathrm{d}\mu}\Big\|_{\mathbb{L}^2(\mu)} \exp\Big(-\frac{Tx^2}{2(\sigma^2(f) + 2C_{\mathrm{P}}\|f\|_\infty x)}\Big),$$

where $\sigma^2(f) = \lim_{t\to\infty} t^{-1}\mathrm{Var}_{\mathbb{P}^\mu}(\int_0^t f(X_s)\,\mathrm{d}s)$

[GGW14] If μ satisfies (LS), $\mu(f) = 0$ and $\mu(\exp(\lambda_\pm f^\pm)) < \infty$ for some $\lambda_\pm > 0$ then we have the (BI)

$$\mathbb{C}_\nu(f, T, x) \leqslant 2\Big\|\frac{\mathrm{d}\nu}{\mathrm{d}\mu}\Big\|_{\mathbb{L}^2(\mu)} \exp\Big(-\frac{Tx^2}{2(\sigma^2(f) + C_{\mathrm{P}}(\Lambda^*)^{-1}(2C_{\mathrm{LS}}/C_{\mathrm{P}})x)}\Big),$$

w. $\Lambda^* = \Lambda^*_+ \vee \Lambda^*_-$ and $\Lambda^*_\pm$ Legendre transf. of $[0, \lambda_\pm] \ni s \mapsto \Lambda_\pm(s) \coloneqq \log\mu(\exp(s(\pm f)))$.

[CG08] If $(\alpha(\mu, \varphi))$ with $\varphi(t) = \mathrm{c}\exp(-t^{\frac{1-q}{1+q}})$, $q \in [0, 1)$ [$q = 0$: exponential mixing, $q \in (0, 1)$: subexponential mixing] and $\|f\|_\infty < \infty$, then for any $x \geqslant C(\mathrm{c}, q)/\sqrt{T}$, it holds

$$\mathbb{C}_\mu(f, T, x) \leqslant 2\exp\Big(-c(q)\Big(\frac{x\sqrt{T}}{\|f\|_\infty}\Big)^{1-q}\Big).$$

Martingale approximation for diffusions

- Let X be a (weak) solution to the SDE

$$\mathrm{d}X_t = b(X_t)\,\mathrm{d}t + \sigma(X_t)\,\mathrm{d}W_t,$$

$b \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}^d;\mathbb{R}^d)$, $\sigma \in \mathrm{Lip}(\mathbb{R}^d;\mathbb{R}^{d\times d})$ and bounded, $a \coloneqq \sigma\sigma^\top$ s.t. $\lambda_-\mathbb{I} \leqslant a(x) \leqslant \lambda_+\mathbb{I}$, $\forall x$

- Let $L = b^\top\nabla + \sum_{i,j} a_{i,j}\partial_{x_i}\partial_{x_j}$ and suppose that for given $f\colon \mathbb{R}^d \to \mathbb{R}$ the Poisson equation $Lg = f$ has some sufficiently regular solution $L^{-1}[f]$
- By Itō's formula: $L^{-1}[f](X_t) - L^{-1}[f](X_0) = \int_0^t LL^{-1}[f](X_s)\,\mathrm{d}s + \int_0^t (\nabla L^{-1}[f](X_s))^\top\sigma(X_s)\,\mathrm{d}W_s$ and hence

$$\int_0^t f(X_s)\,\mathrm{d}s = \underbrace{\int_0^t (-\nabla L^{-1}[f](X_s))^\top\sigma(X_s)\,\mathrm{d}W_s}_{\text{(loc.) martingale}} + \underbrace{L^{-1}[f](X_t) - L^{-1}[f](X_0)}_{\text{remainder}}$$

$\rightsquigarrow$ If we have some control on $L^{-1}[f], \nabla L^{-1}[f]$ we can use martingale approximation for derivation of concentration bounds

- employed in case $d=1$ for exponentially ergodic diffusions in [AWS21; GP07] and for $d \geqslant 1$ and periodic drift [NR20] in the context of drift estimation

Poisson equation under subexponential drift assumptions

Assume $\|b(x)\| \lesssim 1 + \|x\|^{\kappa}$ and for some $q \in (-1, 1)$, $\mathfrak{r}, A > 0$,

$$\langle b(x), x/\|x\| \rangle \leqslant -\mathfrak{r}\|x\|^{-q}, \quad \|x\| > A. \tag{$\mathcal{D}(q)$}$$

[PV01; BRS18] If $\mu(f) = 0$ and $|f(x)| \lesssim 1 + \|x\|^{\eta}$, then for $L^{-1}[f](x) \coloneqq -\int_0^{\infty} P_t f(x)\,\mathrm{d}t$ we have $L^{-1}[f] \in \mathcal{W}^{2,p}_{\mathrm{loc}}(\mathbb{R}^d)$ for any $p > 1$, $L^{-1}[f]$ solves the Poisson equation and

$$|L^{-1}[f](x)| \lesssim 1 + \|x\|^{\eta+1+q}, \quad \|\nabla L^{-1}[f](x)\| \lesssim 1 + \|x\|^{\eta+\kappa+1+q}.$$

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Let $\|\nu\|_f \coloneqq \sup_{|g|\leqslant f} |\nu(g)|$ for some $f \geqslant 1$ and (Ψ_1, Ψ_2) be either pairs of inverse Young functions (i.e., $xy \leqslant \Psi_1^{-1}(x) + \Psi_2^{-1}(y)$) or $(\mathrm{Id}, \mathbf{1})$ or $(\mathbf{1}, \mathrm{Id})$.

Proposition [DFG09; AWST22]

Given $(\mathcal{D}(q))$ we have

$$\|P_t(x,\cdot) - \mu\|_{\mathrm{TV}} \leqslant C(q_+) \exp\left(\iota\|x\|^{1-q_+}\right) \exp\left(-\iota' t^{\frac{1-q_+}{1+q_+}}\right) \quad \text{and} \quad \int_{\mathbb{R}^d} \exp\left(\iota\|x\|^{1-q_+}\right) \mu(\mathrm{d}x) < \infty.$$

Moreover, for $\gamma \geqslant 1+q$, $r_{\gamma,q}(t) \sim (1+t)^{(\gamma-(1+q))/(1+q)}$, $f_{\gamma,q}(x) \sim 1 + \|x\|^{\gamma-(1+q)}$,

$$(\Psi_1(r_{\gamma,q}(t)) \vee 1)\|P_t(x,\cdot) - \mu\|_{\mathbf{1}\vee\Psi_2\circ f_{\gamma,q}} \leqslant C(\Psi)(1 + \|x\|^{\gamma}).$$

Continuous-time concentration result

Theorem [AWST22]

Assume ($\mathcal{D}(q)$), $\|b(x)\| \lesssim 1 + \|x\|^\kappa$ and $|f(x)| \leqslant \mathfrak{L}(1 + \|x\|^\eta)$. Let

$$\rho(\eta, \kappa, q) \coloneqq \begin{cases} 1/(1 - q_+), & \eta = 0 \\ \frac{1}{2} + \frac{\eta + \kappa + 1 + q}{1 - q_+}, & \eta > 0. \end{cases}$$

Then, there exists a constant $\mathfrak{c} > 0$ s.t. for any $x \geqslant 2/\sqrt{T}$,

$$\mathbb{C}_\mu(f, T, x) \coloneqq \mathbb{P}^\mu\left(\left|\frac{1}{T}\int_0^T f(X_t)\,\mathrm{d}t - \mu(f)\right| > x\right) \leqslant \exp\left(-\mathfrak{c}\left(\frac{x\sqrt{T}}{\mathfrak{L}}\right)^{1/\rho(\eta,\kappa,q)}\right).$$

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Poincaré, $\eta = 0$	log-Sobolev, $\eta \leqslant 2$	subexponential, $\eta > 0$
$\frac{\log(1/\delta)}{\varepsilon}$	$\frac{\log(1/\delta)}{\varepsilon}$	$\frac{\log(1/\delta)^{2\rho(\eta,\kappa,q)}}{\varepsilon^2}$

Table 1: Order of sufficient sample length $\Psi(\varepsilon, \delta)$ s.t. (ε, δ)-PAC-bound $\mathbb{P}^\mu(|\mu_T(f) - \mu(f)| \leqslant \varepsilon) \geqslant 1 - \delta$ holds for $T \geqslant \Psi(\varepsilon, \delta)$

Discrete-time concentration result

Let observations $(X_{k\Delta})_{k=1,\dots,n}$ be given for some $\Delta \leqslant 1$. Define $\mathbb{H}_{n,\Delta}(f) \coloneqq \frac{1}{\sqrt{n\Delta}}\mathbb{G}_{n,\Delta}(f)$, where

$$\mathbb{G}_{n,\Delta}(f) \coloneqq \frac{1}{\sqrt{n\Delta}} \sum_{k=1}^{n} f(X_{k\Delta})\Delta.$$

Then for $\mathbb{G}_t(f) \coloneqq t^{-1/2}\int_0^T f(X_t)\,\mathrm{d}t$, $f = \widetilde{f} - \mu(\widetilde{f})$, $\Phi_k(t) \coloneqq \int_t^{k\Delta}(L\widetilde{f}(X_s) - \mu(L\widetilde{f}))\,\mathrm{d}s$, $\omega_k(t) \coloneqq \int_t^{k\Delta} \nabla\widetilde{f}(X_s)^\top \sigma(X_s)\,\mathrm{d}W_s$,

$$\sqrt{n\Delta}(\mathbb{G}_{n,\Delta}(f) - \mathbb{G}_{n\Delta}(f)) = \mu(L\widetilde{f})\frac{n\Delta^2}{2} + \sum_{k=1}^{n}\int_{(k-1)\Delta}^{k\Delta} \Phi_k(t)\,\mathrm{d}t + \sum_{k=1}^{n}\int_{(k-1)\Delta}^{k\Delta} \omega_k(t)\,\mathrm{d}t.$$

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Theorem [AWST22]

Assume ($\mathcal{D}(q)$), $\|b(x)\| \lesssim 1 + \|x\|^\kappa$, $f \in C^2(\mathbb{R}^d;\mathbb{R})$ s.t. $\|D^k f(x)\| \lesssim 1 + \|x\|^{\eta_k}$, $k = 0,1,2$. Define $\alpha \eqqcolon (\kappa + \eta_1) \vee \eta_2$, and let $\widetilde{\gamma} > 1 + q$, $r > 1$, s.t. $\widetilde{\gamma} - (1+q) > r(\alpha \vee (1+q)/(r-1))$. Then, for $p \geqslant 2$,

$$\|\mathbb{H}_{n,\Delta}(f) - \mu(f)\|_{L^p(\mathbb{P}^\mu)} \leqslant \mathfrak{D}\left(\Delta + \sqrt{\frac{\Delta}{n}}\, p^{\frac{\max\{(\widetilde{\gamma}+2\alpha+1-q_+)/2,\, \eta_1+1-q_+\}}{1-q_+}} + \frac{1}{\sqrt{n\Delta}} p^{\frac{1}{2} + \frac{\eta+\kappa+1+q}{1-q_+}}\right) \coloneqq \Phi(n,\Delta,p),$$

and

$$\mathbb{P}^\mu\left(|\mathbb{H}_{n,\Delta}(f) - \mu(f)| > \mathrm{e}\Phi(n,\Delta,x)\right) \leqslant \mathrm{e}^{-x}, \quad x \geqslant 2.$$

Application

- Langevin diffusion

$$\mathrm{d}X_t = -\nabla U(X_t)\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}W_t,$$

has invariant density $\pi(x) \propto \exp(-U(x)) \rightsquigarrow$ sampling from π by numerical approximation of X, e.g., Euler scheme

$$\vartheta_{n+1}^{(\Delta)} = \vartheta_n^{(\Delta)} - \Delta\nabla U(\vartheta_n^{(\Delta)}) + \sqrt{2\Delta}\xi_{n+1}, \quad \vartheta_0^{(\Delta)} \sim X_0, \quad (\xi_n) \underset{\text{iid}}{\sim} \mathcal{N}(0, \mathbb{I}_d)$$

- abundant literature on sampling preicision in TV or Wasserstein distance for U strongly convex or modifications thereof [Dal17; DK19; DM17; DMM19] $\rightsquigarrow \pi(x)\,\mathrm{d}x$ sub-Gaussian
- Assume instead that for some $q \in (0, 1)$

$$\langle \nabla U(x), x/\|x\| \rangle \geqslant \mathfrak{r}\|x\|^{-q}, \quad \|x\| > A. \qquad (\mathcal{U}(q))$$

$\rightsquigarrow \exists\lambda > 0 : \int_{\mathbb{R}^d} \exp\left(\lambda\|x\|^{\widetilde{q}}\right)\pi(x)\,\mathrm{d}x < \infty \iff \widetilde{q} \leqslant 1 - q$
$\rightsquigarrow$ prototypical example: $\pi(x) \propto \exp(-\beta\|x\|^{1-q})$ outside some ball around the origin

Convergence guarantees

Proposition [AWST22]

Assume ($\mathcal{U}(q)$) and that ∇U is bounded. Let $f \in C^2(\mathbb{R}^d)$ s.t. $\|D^k f(x)\| \lesssim 1 + \|x\|^{\eta_k}$, $k = 0, 1, 2$, and consider the burn-in estimator

$$\mathbb{H}_{n,m,\Delta}(f) \coloneqq \mathbb{H}_{n,\Delta}(f) \circ \theta_m = \frac{1}{n} \sum_{k=m+1}^{n+m} f(X_{k\Delta}).$$

Then we have the following approximation guarantees:

	step length Δ	sample size n	burn-in m
ε-prec. sampling	$\frac{\varepsilon^2}{d(\log(\mathfrak{C}/\varepsilon))^{(1-q)/(1+q)}}$	$\frac{d(\log(\mathfrak{C}/\varepsilon))^{2(1-q)/(1+q)}}{\varepsilon^2}$	—
(ε, δ)-PAC bound	$\frac{(\delta\varepsilon)^2}{d(\log(1/\delta))^{2(\eta_0+(q+3)/2)/(1-q)}}$	$\frac{d\mathfrak{D}^2(\log(1/\delta))^{(4(\eta_0+(q+3)/2))/(1-q)}}{\delta^2\varepsilon^4}$	$\frac{d(\log(1/\delta))^{2(\eta_0+q+2)/(1-q)}}{(\delta\varepsilon)^2}$

Table 2: Order of sufficient sampling frequency Δ, sample size n and burn-in m for (ε, δ)-PAC bounds and sampling within ε-TV margin

Summary

- we provide concentration inequalities for subexponentially ergodic diffusions and polynomially bounded functions given continuous observations
- Concentration inequalities for sampled chains are derived from the continuous observation result
- we demonstrate implications on sufficient sample sizes for MCMC for moderately heavy tailed targets

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- we demonstrate implications on sufficient sample sizes for MCMC for moderately heavy tailed targets

Thank you for your attention!

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